MT3 3.22

Show that for an LRC circuit with a small R, the logarythmic decrement is approximately

$$L.D. = \pi R\sqrt{\frac{C}{L}}$$

According to TM5 (p111), the decrement of motion is

$$\frac{A(t)}{A(t+\tau)} = \frac{e^{-\beta t}}{e^{-\beta(t+\tau)}} = e^{\beta\tau}$$

The logarythmic decrement is $\ln(e^{\beta\tau})$

$$LD = \beta\tau$$

For this circuit, $\beta = \frac{R}{2L}$ and $\omega_N = \sqrt{\frac{1}{CL}}$ and

$$LD = \frac{R}{2L}\frac{2\pi}{\omega_s} = \frac{\pi R}{L}\frac{1}{\sqrt{\omega_N^2-\beta^2}}$$

Since R is small, $\beta \ll \omega_N$ so expand the radical

$$LD = \frac{\pi R}{L}\frac{1}{\omega_N}\left(1-\frac{\beta^2}{\omega_N^2}\right)^{-\frac{1}{2}}$$

$$= \frac{\pi R}{L}\sqrt{\frac{CL}{1}}\left[1 + \cancelto{\approx 0}{\frac{\beta^2}{\omega_N^2}} + \cancelto{\approx 0}{\frac{1}{8}\left(\frac{\beta^2}{\omega_N^2}\right)^2} + \dots\right]$$

$$LD \approx \frac{\pi R}{L}\sqrt{\frac{CL}{1}}$$

$$\boxed{LD \approx \pi R\sqrt{\frac{C}{L}}}$$

TA DA!